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Are inflationary predictions sensitive to very high energy physics?

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Abstract: It has been proposed that the successful inflationary description of density perturbations on cosmological scales is sensitive to the details of physics at extremely high (trans-Planckian) energies. We test this proposal by examining how inflationary predictions depend on higher-energy scales within a simple model where the higher-energy physics is well understood. We find the best of all possible worlds: inflationary predictions are robust against the vast majority of high-energy effects, but *can* be sensitive to some effects in certain circumstances, in a way which does not violate ordinary notions of decoupling. This implies both that the comparison of inflationary predictions with CMB data is meaningful, and that it is also worth searching for small deviations from the standard results in the hopes of learning about very high energies.

Keywords: Cosmology of Theories beyond the SM, Physics of the Early Universe.

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1. Introduction

Decoupling is a double-edged sword. On the one hand it ensures that most of low-energy physics is independent of the details of the unknown physics at higher energies, and so allows a meaningful comparison between experiments and theory before a complete 'theory of everything' is constructed. In retrospect, this was a prerequisite for the success of physics as a discipline, since it allows an understanding of Nature at one scale at a time.

On the other hand, decoupling is a real obstacle in our present situation where we have a candidate theory of everything (string theory) but only have access to experiments at energies much below the fundamental string scale. The falsification of such theories is extremely difficult using any terrestrial experiment we can currently conceive. This has led many to look for ways in which very-high-energy physics might have imprinted itself on cosmological observables which were privy to higher-energy effects at very early times but with consequences which have survived until the present epoch.

A recently much-discussed [1, 2, 3, 4] candidate for such an observable is the temperature anisotropy of the cosmic microwave background (CMB), whose properties are presently being measured with unprecedented accuracy [6, 7]. This particular observable suggests itself as a potential probe of very high energies because these anisotropies appear to be well described as arising from primordial metric perturbations (both scalar and tensor) generated during a much-earlier inflationary phase of the Universe (see e.g. [8]).

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Given that such an inflationary phase must have provided at least 60-70 e-folds of inflation in order to solve the various initial-condition problems of the hot big bang, length scales which are now cosmological in size could easily have been below the Planck length prior to freezing out as they left the horizon during inflation [1]. This idea was first investigated in a stringy/quantum gravity context in [2], which also first pointed out that the effects might be as large as order H/M.

Thus, it is not an outrageous hope to think that trans-Planckian effects could manifest themselves in the CMB, though perhaps in a subtle way. Several examples of exotic physics which can do so are put forward in refs. [1, 2, 3], including new particles having non-Lorentz-invariant dispersion relations, short-distance modifications to quantum-mechanical commutation relations, and the preparation of fields in the various nonstandard vacua (more about which later) that can arise in de Sitter space.

But what of decoupling? Why don't these exotic and hypothetical high-energy phenomena decouple from the lower-energy physics of horizon exit? It has recently been argued [4] that decoupling precludes the physics of a high energy M from contributing effects which are larger than $O(H^2/M^2)$, where H is the Hubble scale when the perturbations of interest leave the horizon during inflation.[1] This makes the effects largely undetectable unless M is very close to H. They reach this conclusion by parameterizing the effects of higher-energy physics at horizon exit using interactions in a low-energy effective lagrangian, as is known to be valid elsewhere in other physical applications.

Motivated by the disagreement between the authors of refs. [1, 2, 3] and [4], we present here a critical analysis of how high-energy effects can affect inflationary predictions. In order to keep our discussion precise we examine a simple model in which the higher-energy physics is well understood, consisting of a single heavy scalar particle. By making the heavy physics mundane and simple, we can explicitly compute the predictions of the high-energy theory and see when an effective description applies. In our model a pre-inflationary period of matter-dominated FRW expansion can occur, which allows us to unambiguously decide what the initial quantum state of the system should be. Apart from these reasons, we also believe it to be useful to know how conventional kinds of high-energy physics can change the predictions for inflation, not least to use as a benchmark against which to compare the more exotic types which have been considered to date.

We find an element of truth in both sides of the above-mentioned dispute. In a nutshell:

- We find that it *is* possible for the high-energy scalar to modify the inflationary predictions for the fluctuations in the CMB, provided the epoch of horizon exit of the relevant wavelengths for the CMB does not occur too much later than the onset of inflation. In fact, by adjusting parameters, we can have observable effects after as many as 30 e-foldings of inflation before horizon exit. The reason this modification occurs is due to a failure of the adiabatic approximation stemming from the rapid oscillations of the heavy field before horizon exit. This then ensures that the epoch before horizon exit will not be well described by a low-energy effective theory. This

[1] As pointed out in [5], there is a class of higher-derivative terms which lead to potentially larger corrections than those described in [4].

shows how the interference with standard inflationary predictions is consistent with decoupling, inasmuch as observers at all times agree on why the high-energy physics does not decouple.

- Although we find that the heavy scalar can influence inflationary predictions, the desired effect is only important in comparatively special parts of parameter space or for a specific class of initial conditions. The *generic* situation is to have the high-energy scalar decouple, and so not alter the standard inflationary predictions. In particular all deviations from standard inflationary predictions can be made negligible by having sufficient inflation occur before horizon exit, at least for the conventional heavy physics considered here.

Our results also bear on a related issue which has arisen within the discussion of trans-Planckian physics. The question is whether it is *in principle* sensible to use the various nonstandard vacua of de Sitter space. In particular, various recent calculations have been used to argue that some of these vacua give physically nonsensical results [9]. We show here that it is possible to obtain some of the non-standard vacua starting from the usual vacuum in a pre-inflationary phase of matter- or radiation-dominated FRW expansion. This possibility shows that these particular inflationary vacua must make physical sense, since they arise by time evolving a pre-existing well-behaved state. The reasonable states which are obtained in this way differ from those discussed in [9] in that they approach the standard (Bunch-Davies [10]) vacuum for sufficiently high momenta. Together, these results make it unlikely that reasonable high-energy physics can lead to nonstandard vacua for modes of arbitrarily high momentum. For lower momenta, we show that it is always possible to choose a momentum-dependent nonstandard vacuum in such a way as to reproduce in the CMB fluctuations the effects we find from a heavy scalar field.

All told, we find our results to be very encouraging since they suggest we may be able to have our cake and eat it too. On the one hand we can sensibly compare standard inflationary results with CMB measurements, secure in the knowledge that the comparison is independent of the vast majority of things which could be happening at higher energies. On the other hand there are specific kinds of high-energy physics which *can* leave their imprint in the CMB, and for which searches should therefore be made. In particular we might learn something about how long inflation had gone on before observed scales left the horizon. The resulting implications for the CMB might indeed be observable in the next generation of experiments, such as MAP [11] or PLANCK [12].

We present our discussion in the following way. Section (2) describes our model of high-energy physics. It consists of a standard hybrid-inflation model [13] containing a light inflaton, ϕ, together with a much heavier scalar, χ. We then examine how the heavy scalar field evolves if it is not started at a local minimum of its potential, and how sufficiently large oscillations of this scalar can dominate the universe's energy density, causing a period of pre-inflationary matter domination. The implications of these oscillations for inflaton fluctuations are described analytically and numerically in section (3), ending with the consequences which they imply for post-inflationary density fluctuations. Section (4) considers an alternative model, for which observable implications persist for a longer period during

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inflation, and for which the visible signatures might occur at smaller angular scales than in the previous case. Our conclusions are briefly summarized in section (5).

2. The model

For the purposes of our analysis we initially adopt a textbook [8] hybrid-inflation model [13], defined by the lagrangian density

$$-\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi + V(\phi,\chi)\right], \tag{2.1}$$

with

$$V(\phi,\chi) = \frac{1}{2}m^2\,\phi^2 + \lambda\left(\chi^2 - v^2\right)^2 + \frac{1}{2}g\,\chi^2\phi^2 + \tilde{\lambda}\,\phi^4\,. \tag{2.2}$$

The potential has absolute minima at $\chi = \pm v$ and $\phi = 0$, but also has a long trough at $\chi = 0$ provided $g\,\phi^2 > \lambda\,v^2$. In a later section (3.3) we will consider a variation of this model with a trilinear coupling, $g'\chi\phi^2$, which will be shown to have a qualitatively different effect on the CMB temperature anisotropy.

2.1 Conditions for inflation

In the model (2.1) the inflaton, ϕ, starts at $\phi = \phi_0$, with $g\phi_0^2 \gg \lambda\,v^2$, and then slow-rolls along the $\chi = 0$ trough. To ensure that the trough is sufficiently flat, we require the following conditions:

1. In order to ensure that $V(\phi,\chi=0) \approx \frac{1}{4}\lambda\,v^4$ for all $\phi < \phi_0$ we require $m^2\,\phi_0^2 \ll \lambda\,v^4$, as well as $\tilde{\lambda} \ll m^2/\phi_0^2$ (which allows us to neglect $\tilde{\lambda}$ in what follows). This is consistent with having sufficiently large ϕ_0 provided $m^2 \ll g\,v^2$. Under these assumptions the Hubble scale during inflation is $H^2 \approx \lambda\,v^4/(12\,M_p^2)$, where $1/M_p^2 = 8\pi G$. In order to trust Einstein's equations we take $\lambda\,v^2 \ll M_p^2$, and so have $H^2 \ll v^2$.

2. Define the slow-roll parameters [8] as $\epsilon = (M_p\,V'_{\rm tr}/V_{\rm tr})^2$ and $\eta = M_p^2\,V''_{\rm tr}/V_{\rm tr}$, where the truncated scalar potential is $V_{\rm tr}(\phi) = V(\phi,\chi=0)$. By requiring $4m^2M_p^2 \ll \lambda\,v^4$, or equivalently, $m^2 \ll 3\,H^2$ we can make the slow-roll parameters small enough for sufficient inflation.

Under these conditions the field χ is a heavy degree of freedom throughout all but the very end of the inflationary epoch. Its mass is given by the curvature of the potential transverse to the trough, which depends on the value of the inflaton field through the relation

$$M^2 = \left.\frac{\partial^2 V}{\partial\chi^2}\right|_{\chi=0} = -\lambda\,v^2 + g\,\phi^2 \approx g\phi^2\,. \tag{2.3}$$

The last approximation follows right up until the end of inflation by virtue of the conditions described above. We can nevertheless follow the evolution of this scalar using Einstein's equations during an inflationary phase provided $\lambda\,v^2 \ll M^2 \ll M_p^2$. If the couplings g and λ are not too different from one another and are not too small, we also have $m \ll H \ll M$ during inflation.

2.2 χ oscillations and a possible pre-inflationary phase

The picture so far is standard. Our only modification will be to choose χ to lie at $\chi_0 \neq 0$, with $\dot{\chi}_0 = 0$ initially, rather than at the bottom of the trough, $\chi_0 = 0$. (See [14] for other discussions of the initial conditions in hybrid inflation models.) Since large enough χ oscillations can dominate the energy of the universe, in this section we do not assume that the Hubble scale $H = \dot{a}/a$ is a constant.

If χ_0 is chosen close enough to the trough's bottom (*i.e.* with $\lambda\chi_0^2 \ll M^2$) then we may neglect the effect of the χ^4 terms in the action on the χ field equations, leading to

$$\ddot{\chi} + 3H\,\dot{\chi} + M^2(\phi)\,\chi \approx 0\,. \tag{2.4}$$

As usual, we consider only the evolution of the homogeneous χ mode, i.e. we take $\nabla\chi = 0$, since spatially-inhomogeneous modes will be redshifted as inflation proceeds.

A general solution to this equation may be obtained, regardless of the time-dependence of H, so long as we may neglect $\dot{\phi}/\phi$ and H in comparison with M. (We have already seen this neglect to be justified during the inflationary regime which is of most interest for the purposes of this paper.) Assuming the initial condition $\dot{\chi}_0/\chi_0 \lesssim O(H)$ we find

$$\chi(t) \approx A(t)\cos\Big[M(\phi)(t-t_0)\Big]\,, \tag{2.5}$$

where the slowly-varying envelope is given by $A(t) = \chi_0\,[a(t_0)/a(t)]^{3/2}$, and $M^2(\phi) \simeq g\phi^2$. This evolution describes a fast oscillation rather than a slow roll because of the condition $M \gg H$, whose origin is discussed above.

The energy density associated with these oscillations is

$$\rho_\chi(t) = \frac{1}{2}\Big(\dot{\chi}^2 + M^2\,\chi^2\Big) = \frac{1}{2}M^2A^2(t) = \frac{1}{2}\,M^2\,\chi_0^2\,\left(\frac{a(t_0)}{a(t)}\right)^3, \tag{2.6}$$

which scales with $a(t)$ as does non-relativistic matter. If the initial amplitude is chosen to be large enough this energy can be larger than $\frac{1}{4}\lambda\,v^4$, in which case the universe is initially matter-dominated. The amplitude of the χ oscillations is then damped by the universal expansion, and so long as the ϕ roll remains slow an inflationary phase eventually begins. As the χ oscillations damp to zero we match onto the standard picture of hybrid inflation based on the slow ϕ roll.

In figures 1, 2 we plot the total energy density and the ϕ kinetic energy for several such rolls, calculated numerically, where the initial condition ϕ_0 is fixed but χ_0 varies. The plot shows how the length of the inflationary period shrinks as χ_0 gets larger, corresponding to the longer time required to damp away the energy in χ oscillations.

From these considerations we see that there are potentially three times we should keep track of:

- We define $t = 0$ as the time during inflation when inflaton fluctuations having $k = k_{\rm COBE} \cong 7.5 H_{\rm present}$ [15] leave the horizon: $k_{\rm COBE}/a_{he} = H$. Here $H_{\rm present}$ denotes the present-day Hubble scale. We denote the size of the oscillatory envelope at this time by $\chi_{he} = A_{he} \equiv A(t=0)$. For later convenience we normalize the scale factor so that $a_{he} \equiv a(t=0) = 1$.

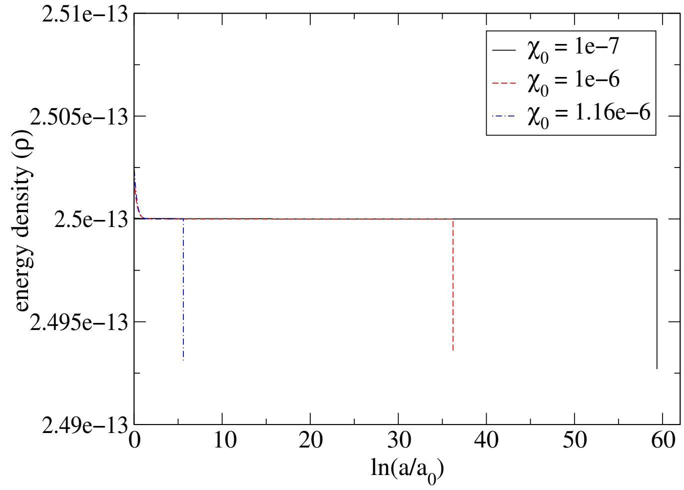

Figure 1: Energy density of the universe in units of M_p^4 for 3 different initial conditions in χ_0, while ϕ_0 and all the parameters of the potential are fixed. Here, we have: $\lambda = g = 1$, $v = 0.001 M_p$, $m = 1.1121 \times 10^{-7} M_p$ and $\phi_0 = 0.01945 M_p$.

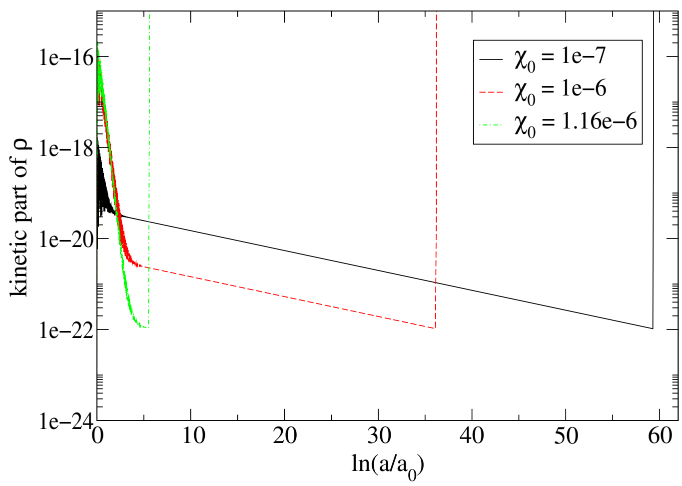

Figure 2: The kinetic energy of the ϕ roll for the same parameters as in the previous figure.

- $t = t_0 < 0$ denotes the time when χ oscillations begin, which may or may not lie within a pre-inflationary oscillation-dominated phase. As in previous discussions $\chi_0 = A_0$ and a_0 denote the values of the χ field envelope and the scale factor at this time.

- If there is a pre-inflationary phase then t_i, with $t_0 < t_i < 0$, denotes the time where inflation begins, as defined by when the energy in χ oscillations falls below the constant scalar potential value $\frac{1}{4}\lambda v^4$. A subscript 'i' then denotes the value taken by a quantity at $t = t_i$. If the initial χ oscillations are never large enough to dominate the energy density, we need never consider t_i, and we instead define $t_i = t_0$ and use these two symbols interchangeably.

Since the amplitude of oscillations during inflation is exponentially damped, $\chi_i =$

$\chi_{he} e^{3H|t_i|/2}$, the time t_i is related to the size, χ_{he} of the oscillation envelope at horizon exit. To see this notice that χ_i is found by equating the χ-oscillation energy at t_i, $\frac{1}{2} M^2 \chi_i^2$ to the inflationary vacuum energy, $\frac{1}{4} \lambda v^4 = 3H^2 M_p^2$, giving

$$\chi_i^2 = \frac{6H^2 M_p^2}{M^2} . \tag{2.7}$$

The number of e-foldings between the beginning of inflation and horizon exit then is

$$H|t_i| \sim \frac{1}{3} \ln \left(\frac{\chi_i^2}{\chi_{he}^2} \right) \sim \frac{1}{3} \ln \left(\frac{6M_p^2/M^2}{\chi_{he}^2/H^2} \right) . \tag{2.8}$$

Clearly — for fixed fluctuation size, χ_{he}/H — the later horizon exit occurs after the onset of inflation, the lower M must be, and hence the smaller m, v and H must also be in order to have sufficient inflation *after* horizon exit.

If $t_0 < t_i$ then the initial amplitude must be larger still, by a matter-dominated expansion factor of $[a(t_0)/a(t_i)]^{3/2} = t_0/t_i$. The duration of the matter-domination oscillations also cannot be too long without pushing the initial oscillation energy above the Planck scale, and so invalidating the use of Einstein's equations. This condition requires

$$\frac{t_0}{t_i} < \frac{M_p}{\sqrt{3}H} , \tag{2.9}$$

which can be enormous, for small H.

3. Modifications to inflaton fluctuations

We now turn to a computation of how the χ oscillations change the power spectrum of inflaton fluctuations which get imprinted onto the CMB. For simplicity, we first consider the case where the χ oscillations occur entirely during the inflationary phase ($t_i \sim t_0$).

Consider the quantum fluctuations of the inflaton, φ. A mode with wave number k has the equation of motion

$$\ddot{\varphi}_k + 3H\dot{\varphi}_k + \left[k^2 e^{-2Ht} + V''(\phi) + g\chi^2(t)\right] \varphi_k = 0 . \tag{3.1}$$

In the absence of the coupling to χ, and taking $V'' \ll H^2$ as required for slow roll, the standard positive and negative frequency solutions are

$$\varphi_k^{\pm} = \left(\pm iH + \frac{k}{a} \right) e^{\pm ik/aH} , \tag{3.2}$$

where the scale factor is $a = e^{Ht}$. In the standard canonical quantization, it is assumed that the universe starts in the adiabatic vacuum state which is annihilated by a_k in the mode expansion

$$\phi \sim \sum_k \left(a_k \varphi_k^+ + a_k^\dagger \varphi_k^- \right) . \tag{3.3}$$

This is also known as the Bunch-Davies vacuum, $a_k|BD\rangle = 0$. If the universe started in a different state $|\alpha_k\rangle$, annihilated by some linear combination of a_k and $a_k^\dagger$, say $b_k =$

$\cosh\alpha_k\, a_k + e^{-i\delta}\sinh\alpha_k\, a_k^\dagger$ [17], then the mode expansion would be in terms of functions $\widetilde{\varphi}_k^+$, $\widetilde{\varphi}_k^-$ given as linear combinations of the positive and negative frequencies,

$$\phi \sim \sum_k \left(b_k \widetilde{\varphi}_k^+ + b_k^\dagger \widetilde{\varphi}_k^- \right),$$
$$\widetilde{\varphi}_k^+ = \cosh\alpha_k\, \varphi_k^+ + e^{i\delta}\sinh\alpha_k\, \varphi_k^- . \tag{3.4}$$

It is the interference between positive and negative frequency components of $\widetilde{\varphi}_k^+$ which gives rise to potentially observable effects in the CMB power spectrum, since the latter is derived from $|\widetilde{\varphi}_k^+|^2$ evaluated at horizon crossing, $k/a = H$.

If $\delta = 0$, the effect on the power spectrum is a modulation of the standard one,

$$\widetilde{P}_k \sim \lim_{t\to\infty} |\widetilde{\varphi}_k|^2 = |\cosh\alpha_k - \sinh\alpha_k|^2 P_k\,, \tag{3.5}$$

where P_k is the standard Harrison-Zeldovich spectrum.

The simple point we wish to make here is that the coupling of ϕ to χ can also induce an admixture of positive- and negative-frequency components in solutions for ϕ which are initially purely positive frequency. In the next section we demonstrate this explicitly by numerically evolving the fields forward in time starting from various initial conditions for χ.

3.1 Asymptotic forms

Before describing these numerical results, we first identify some of their limiting features by considering an approximate analytic solution.

In the classical analysis of interest here, the influence of χ oscillations on ϕ fluctuations can be obtained explicitly by using the solution $\chi(\phi, t)$ from eq. (2.5) in the mode equation (3.1). The mode equations can then be solved perturbatively by use of the appropriate Greens' function, which since we are interested in the classical evolution of the fluctuations, is the retarded one. It is defined by

$$G_{k,<}(t,t') = \begin{cases} 0\,, & t < t' \\ \varphi_k^+(t)\varphi_k^-(t') - \varphi_k^-(t)\varphi_k^+(t')\,, & t > t' \end{cases} \tag{3.6}$$

and obeys the differential equation

$$\left[\partial_t^2 + 3H\partial_t + k^2 e^{-2Ht}\right] G_{k,<}(t,t') = -2ik^3 e^{-3Ht}\delta(t-t')\,. \tag{3.7}$$

The perturbed positive frequency solution of eq. (3.1) is

$$\widetilde{\varphi}_k^+(t) \equiv \varphi_k^+(t) + \delta\varphi_k^+(t)\,,$$
$$\delta\varphi_k^+(t) \cong \int_{t_0}^t dt'\, G_{k,<}(t,t') \frac{e^{3Ht'}}{2ik^3}\, gA^2(t')\, \cos^2[M(t'-t_0)]\varphi_k^+(t')\,, \tag{3.8}$$

and we assume for simplicity no pre-inflationary period, so $t_i = t_0$. Direct comparison of eqs. (3.8) and (3.6) shows that the perturbed solution has an admixture of the original positive- and negative-frequency modes.

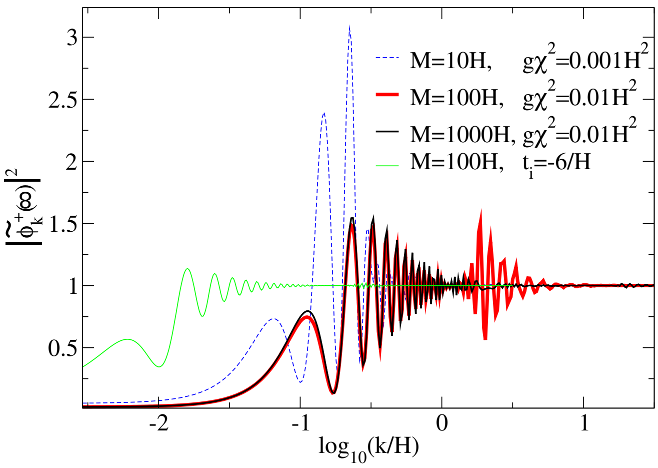

Figure 3: $|\tilde{\phi}_k^+(\infty)|^2$ in units of H^2 for the hybrid inflation model, as a function of $\log_{10}(k/H)$ for several values of M and $g\chi^2_{\rm he}$, rightmost three curves for $t_0 = t_i = -4/H$. Leftmost curve shows the effect of taking an earlier initial time, $t_i = -6/H$, with $g\chi^2_{\rm he} = 10^{-6}$.

Since we are interested in the amplitude of the perturbations well after their oscillations have frozen out, we may take $t \to \infty$ and use eq. (3.5) to compute the change in the spectrum of fluctuations:

$$\frac{\delta P}{P}(k) = \left| \frac{\tilde{\varphi}_k^+(\infty)}{\varphi_k^+(\infty)} \right|^2 - 1 \approx \frac{2{\rm Im}\,\delta\varphi_k^+(\infty)}{H}\,, \tag{3.9}$$

where we use the unperturbed result: $\varphi_k^+(\infty) = iH$.

We can use the perturbative expression, eq. (3.8), to deduce useful asymptotic expressions for the solution in the limit of large and small k. Changing variables to $u = (k/H)\,e^{-Ht}$, the large time limit corresponds to $u \to 0$, and the integral can then be rewritten as

$$\delta\varphi_k^+(\infty) = \frac{H^2}{k^3}\, g\chi^2_{he} \int_0^{u_0} \frac{du}{u}\, \cos^2\left(\frac{M}{H}\ln\frac{u}{u_0}\right) e^{iu}(i+u)\,(\sin u - u\cos u)\,, \tag{3.10}$$

where $u_0 = (k/H)\,e^{-Ht_0}$ and we use $a_{he} = 1$. Taking first as $k \to 0$ (hence $u_0 \to 0$), the integral can be evaluated to give:

$$\lim_{k\to 0} \delta\varphi_k^+(\infty) \approx -i\,\frac{g\chi^2_{he}}{18H}\, e^{3H|t_0|} = -i\,\frac{g\chi_0^2}{18H}\,, \tag{3.11}$$

up to $O(H^2/M^2)$ corrections. Comparing this to the unperturbed value $\varphi_k^+(\infty) = iH$, we see that this causes a suppression of power which depends on the initial fluctuation amplitude, χ_0, and is independent of M in the limit $M \to \infty$.

This loss of power at long wavelengths is also seen in the full (nonperturbative) numerical integration of the equation of motion (3.1), whose results are shown in figure 3. Notice in these numerical examples that the deviation of $|\tilde{\phi}_k^+(\infty)|$ from its normal value (H) can be large, even if χ_{he} is small. A flavor of this also seen in the perturbative expression due to the conversion of χ_{he} to χ_0 by the factor $e^{|3Ht_i/2|}$.

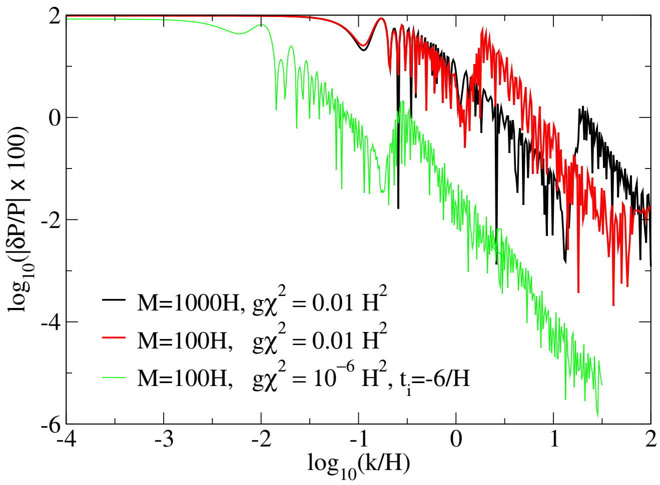

Figure 4: Log of absolute value of percent deviation of power spectrum as a function of $\log_{10}(k/H)$, for $M = 100H$ and $M = 1000H$, with $t_0 = t_i = -4/H$ and $g\chi^2_{\rm he} = 0.01H^2$, and $M = 100H$ with $t_i = -6/H$ and $g\chi^2_{\rm he} = 10^{-6}H^2$. Notice that the deviation is large at low k not because the power is large, but rather because it is smaller than normal. Order of curves in legends coincides with that at right hand edge of the graph.

Equation (3.10) also gives the limiting form when $k \to \infty$ ($u_0 \to \infty$), although this limit is somewhat more difficult to obtain. Since eq. (3.9) shows we are only concerned with the imaginary part of $\delta\varphi^+_k(\infty)$ for the power spectrum we focus on this. In the limit of large M and large u_0 we find

$$\lim_{k\to\infty} \mathrm{Im}\,\delta\varphi^+_k(\infty) = \frac{g\chi^2_{he}M}{4k^2}e^{H|t_0|}f(k) = \frac{g\chi^2_0 M}{4k^2}e^{-2H|t_0|}f(k)\,, \tag{3.12}$$

where $f(k)$ is an oscillatory function of k of unit amplitude. Thus the deviations from the standard power spectrum are oscillatory with an amplitude that falls like $1/k^2$ for large k. This behavior is also confirmed by the numerical results, as shown in figure 4.

We can understand the frequency of the fast oscillations in the numerical results as coming directly from those of the χ field itself. The numerics indicate that this frequency is the same as that of the function $\cos[(2M/H)\ln(k)]$. If we rewrite k in terms of the physical wavelength of the mode, $k = k_{\rm phys}e^{-Ht}$, the previous function becomes $\cos(2Mt)$ up to a phase. This is what we expect for δP due to the driving field, χ, as is seen using the identity $\cos^2(Mt) = \frac{1}{2}[1 + \cos(2Mt)]$.

The fact that t_0 only enters the perturbative results through the combination $u_0 = (k/H)\,e^{-Ht_0}$ means that the effect of choosing earlier initial times, t_0, to be more negative is largely to translate the graphs to the left in $\log(k)$ space. At the same time the normalization changes because of our convention of using $\chi_{\rm he} = e^{3H|t_i|/2}\chi_0$ as the input parameter. These features are also borne out by the numerics.

3.2 Implications for the CMB

To quantify the effect of the oscillating scalar on the CMB fluctuations, we have integrated the equation of motion for inflaton fluctuations numerically for a large range of parameters. We then compute the fractional deviation in the power spectrum using eq. (3.9).

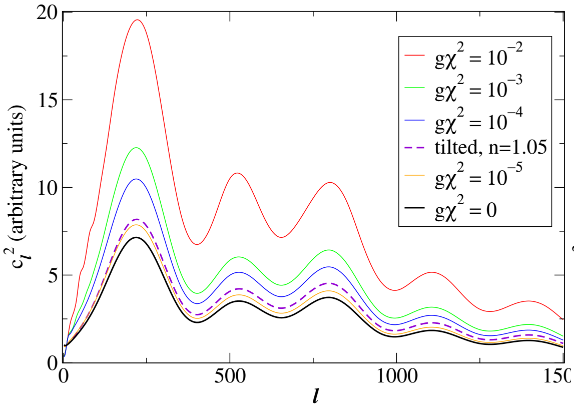

Figure 5: Solid lines: Doppler peaks using successively smaller values of the initial χ amplitude. Dashed line: effect of a tilted spectrum with index $n = 1.05$, and no coupling between χ and ϕ. The legend gives $g\chi_{he}^2$ in units of H^2.

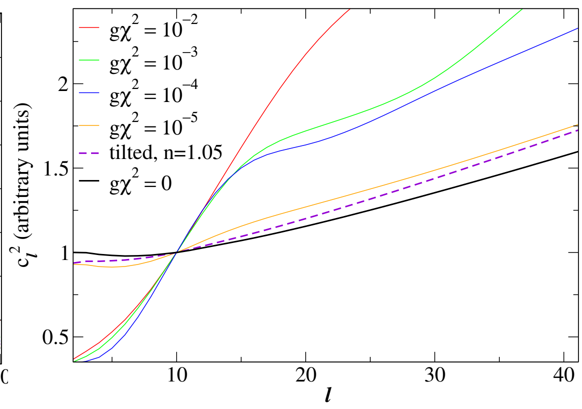

Figure 6: The same as in figure 5, showing close-up of the low-l modes. The order of curves in the legend is the same as in the right hand side of the figure.

In figure 4 we plot the log of the absolute value of the percentage deviation ($\log_{10}(|\delta P|/P \times 100)$) as a function of $\log_{10}(k/H)$, for a range of values of M, and for two different values of t_0. We find a spectrum of deviations which oscillates rapidly, due to the fast χ oscillations, with an envelope which interpolates between the large- and small-k limits described above.

As can be seen from figures 3 and 4, the largest deviations in the primordial power spectrum occur at the lowest k values. This can be understood because at early times the average value of $g\chi^2$ is potentially large, and thus effectively gives the inflaton a significant mass. This in turn gives rise to a tilted spectrum at low values of k, which is constrained by measurements of the large-angle CMB anisotropy. We have computed the effect of input spectra like these on the CMB using CMBFAST [18], with the results shown in figures 5 and 6. We have chosen to normalize all the spectra to a common value (unity) at $l = 10$, and so the depression in fluctuations on very large scales leads to an enhancement of the size of the first and subsequent peaks.

For comparison, the effect of tilting the input spectrum with a spectral index of 1.05, and no coupling to $\chi(t)$, is shown as the dashed line in these figures. This shows that the effect of χ is qualitatively close to that of tilt, but still in principle distinguishable. In particular, the anisotropy of the tilted model falls below that of the $g\chi_{he}^2 = 10^{-5}$ model for $10 < l \lesssim 70$, but becomes larger for $l \gtrsim 70$.

3.3 Phenomenology

Now that we know how big χ oscillations must be at horizon exit in order to be detected we may ask how natural or fine-tuned our initial conditions must be in order to affect the CMB in an observable way. Since figures 5 and 6 show that an amplitude $g\,\chi_{he}^2 \sim 10^{-5}\,H^2$ has implications which are comparable to a 5% tilt to the Zeldovich spectrum, we take this as our benchmark for what an observable deviation would be in the predictions for CMB fluctuations.

Since the amplitude of oscillations during inflation is exponentially damped, $\chi_i = \chi_{he} e^{3H|t_i|/2}$, χ_{he} will be unobservable if too much time passes between the onset of inflation and the time of horizon exit. From eq. (2.8) we see this implies

$$H\,|t_i| \lesssim \frac{1}{3}\,\ln\left(\frac{\chi_i^2}{\chi_{he}^2}\right) \sim \frac{1}{3}\,\ln\left(\frac{6\times 10^5\, g M_p^2}{M^2}\right). \tag{3.13}$$

To see how large this can be, we must insure that the rate of energy loss of the χ condensate due to inflaton production is not faster than its decay due to Hubble damping. We will estimate this rate as $\Gamma = n\langle \sigma v\rangle$, where σ is the cross section for $\chi\chi \to \phi\phi$ and the condensate is regarded as a collection of χ quanta at rest (notice that the product σv is well-defined in the limit $v \to 0$ since the cross section has a factor of $1/v$). The density of these quanta is ρ/M, where their energy density is $\rho = M^2\chi^2$. The annihilation cross section can be estimated as $\sigma v \sim \frac{g^2}{64\pi^2}(2M)^{-2}$. Demanding that $\Gamma < H$ at the initial time (which insures it will be true also at later times), we get an upper limit on the coupling strength,

$$g^2 < \frac{256\pi^2 M^2}{\sqrt{3}\chi_i M_p} \cong \frac{256\pi^2 M^3}{3HM_p^2} \tag{3.14}$$

where in the last step we used the largest value of χ_i consistent with inflaton domination of the energy density. Choosing all parameters optimistically we could have $g \cong 0.7$, $\lambda \sim O(1)$ and $H \sim 100\,\mathrm{GeV}$, in which case $v \sim (HM_p)^{1/2} \sim 10^{10}\,\mathrm{GeV}$. Taking then $M \sim 10^{12}\,\mathrm{GeV}$ would give $H|t_i| \sim 12$, so horizon exit can occur quite deep (more than 10 e-foldings) into the inflationary phase.

In summary, we conclude that in this model if inflation persists for more than about 10 e-foldings longer than the minimum necessary to solve the usual problems of big bang cosmology, none of the effects which we are considering here are likely to be observable. However rapid χ oscillations can significantly extend the period of inflation before horizon exit to which CMB observations could be sensitive. The strongest effects on the CMB anisotropy appear at low l values, and so would not be expected to be discovered by experiments whose advantage is their smaller angular resolution. This CMB window onto high-energy physics due to non-adiabatic oscillations is perhaps no less interesting than the more nebulous possibility of observing the effects of trans-Planckian physics.

4. An alternative model

For comparison we now turn to a slightly different model, for which the observable window is wider and has qualitatively different features. The model is motivated by the observation that roughly twice as many e-foldings would be possible if the χ-ϕ coupling were linear in χ instead of quadratic: i.e. if $\mathcal{L}_{\chi\phi} = g'\,\chi\,\phi^2$. In this case the amplitude of χ oscillations still fall like $a^{-3/2}$, but this now implies that the perturbations, δP_k, to the CMB fluctuations are only damped as $a^{-3/2}$ instead of as a^{-3}. Equally interesting is that in this case the distortions of the primordial power spectrum can have a maximum at some intermediate value of the wave number, which could correspond to multipoles of the CMB anisotropy beyond the first Doppler peak.

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We consider, then, a model in which the inflaton field starts near $\phi = 0$ and rolls away from the origin during inflation, such as the inverted hybrid inflation model [16]. In this case χ is no longer the field which controls the duration of inflation, whose existence is not important for the present discussion; rather it is another field which is introduced just to obtain the new kinds of effects we are interested in exploring. The lagrangian can be written as

$$V(\phi, \chi) = v^4 - \frac{1}{2} m^2 \phi^2 + \frac{1}{2} M^2 \chi^2 + g' \chi \phi^2 . \tag{4.1}$$

Here the coupling g' has dimensions of mass. The mass parameter M in this model does not depend on ϕ, and we must have $M^2 \gg H^2$ to ensure that χ is not slowly rolling. The term v^4 denotes the constant energy density which dominates during inflation.

In the limit that the inflaton rolls very slowly so that $\dot{\phi}$ can be neglected, the exact solution for $\chi(t)$ is:

$$\chi(t) = \frac{g'\phi^2}{2M^2} + e^{-3H(t-t_0)/2} \left(\chi_0 - \frac{g'\phi^2}{2M^2} \right) \cos \left[\widetilde{M}(t - t_0) \right] . \tag{4.2}$$

where $\widetilde{M} = \sqrt{M^2 - \frac{9}{4} H^2}$. Since we are interested in the case $M \gg H$, we can take $\widetilde{M} \cong M$.

We need to keep the inflaton nearly massless relative to H to insure that it rolls slowly enough to get sufficient inflation. Since the constant term in $g'\chi$ is the inflaton mass squared, this requires $g'^2 \phi^2 / M^2 \ll H^2$. We also demand that the χ energy density is subdominant relative to that of the inflaton; otherwise we initially have a matter dominated era rather than inflation (a possibility which we will discuss below). Moreover we are assuming that $\phi \cong 0$ during inflation, so that the solution for χ in eq. (4.2) can be written as:

$$\chi(t) \simeq e^{-3Ht/2} \chi_{he} \cos \left(M(t - t_0) \right) , \tag{4.3}$$

where $\chi_{he} = \chi_0 \, e^{3Ht_0/2}$.

The calculation of the power spectrum proceeds much as before, and we only record here those expressions which differ from the previously-explained hybrid-inflation example. Neglecting the inflaton mass, the equation governing the quantum fluctuations of the inflaton, φ becomes

$$\ddot{\varphi}_k + 3H\dot{\varphi}_k + \left[k^2 e^{-2Ht} - g'\chi(t) \right] \varphi_k = 0 , \tag{4.4}$$

which generates the same retarded Green's function as before.

The perturbed positive-frequency solution of eq. (3.1) is then

$$\begin{aligned} \widetilde{\varphi}_k^+(t) &\equiv \varphi_k^+(t) + \delta\varphi_k^+(t) , \\ \delta\varphi_k^+(t) &\cong \int_{t_0}^t dt' \, G_{k,<}(t,t') \frac{e^{3Ht'}}{-2ik^3} \, g' A(t') \, \cos[M(t' - t_0)] \, \varphi_k^+(t') . \end{aligned} \tag{4.5}$$

The limiting forms of eq. (4.5) may be deduced as before, by changing variables to $u = (k/H)e^{-Ht}$. This time we find

$$\delta\varphi_k^+(\infty) = -\frac{H^{1/2}}{k^{3/2}} \, g' A_{he} \int_0^{u_0} \frac{du}{u^{5/2}} \, \cos\left(\frac{M}{H} \ln \frac{u}{u_0} \right) e^{iu} (i + u) \left(\sin u - u \cos u \right) , \tag{4.6}$$

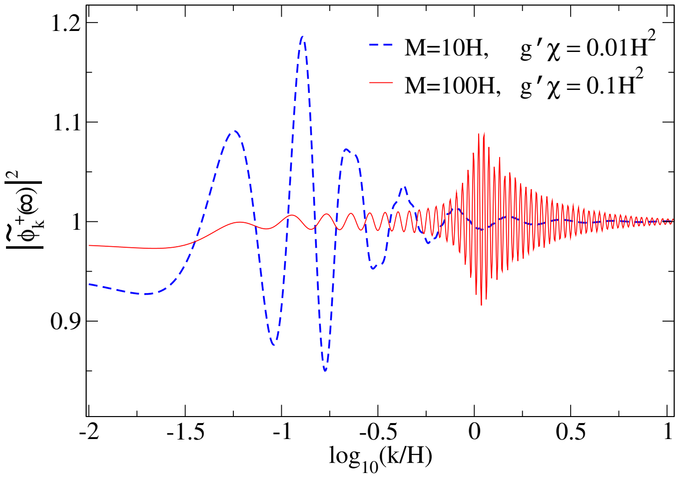

Figure 7: $|\tilde{\phi}_k(\infty)|^2$ in units of H^2 as a function of $\log_{10}(k/H)$ in the trilinear coupling model, for several values of M and $g\chi_{\rm he}^2$.

for which the $k \to 0$ limit then becomes

$$\lim_{k\to 0} \delta\varphi_k^+(\infty) = -i\,\frac{g' A_{he} H}{2M^2}\, e^{3H|t_0|/2} = -i\,\frac{g' A_0 H}{2M^2}\,. \tag{4.7}$$

As was true in the previous example, the $k \to 0$ limit of $\delta\varphi_k^+(\infty)/H$ is a constant, which is proportional to $g'\,A_0/M^2$. However in contrast to the previous model, this deviation vanishes in the limit of large M, so that power at large wavelengths will be relatively unchanged.

The limiting behavior as $k \to \infty$ ($u_0 \to \infty$) differs from our previous example, since the integral converges for large u_0. We find in the large-k limit:

$$\lim_{k\to\infty} \delta\varphi_k^+(\infty) \cong 0.5\,\frac{g' A_{he} H^{1/2}}{k^{3/2}} \times (\text{oscillatory function}) \tag{4.8}$$

whose amplitude is independent of M. These limits are borne out by our numerical results, presented in figures 7–9, which approach an M-dependent constant for small k and fall with an M-independent envelope for large k. The numerical constant in eq. (4.8) is established by comparison with these numerical results.

Figures 8, 9 plot the log of the percentage deviation ($\log_{10}(\delta P/P \times 100)$) as a function of $\log_{10}(k/H)$, for a range of values of M, and for two different values of t_0. The spectrum of deviations is observed to reach a maximum for wave numbers of order

$$k_{\rm max} \cong \frac{M}{2} e^{-H|t_0|} \tag{4.9}$$

before joining the universal $k^{-3/2}$ fall-off at larger k. From these two results we read off the maximum size of the deviation as being:

$$\frac{\delta P}{P}(k_{\rm max}) \cong \frac{g' A_{he}\, e^{3H|t_0|/2}}{H^{1/2}(M/2)^{3/2}} = \frac{g' A_0}{H^{1/2}(M/2)^{3/2}}\,. \tag{4.10}$$

This maximum value is therefore greater than the small-k limit by the factor $(M/H)^{1/2}$.

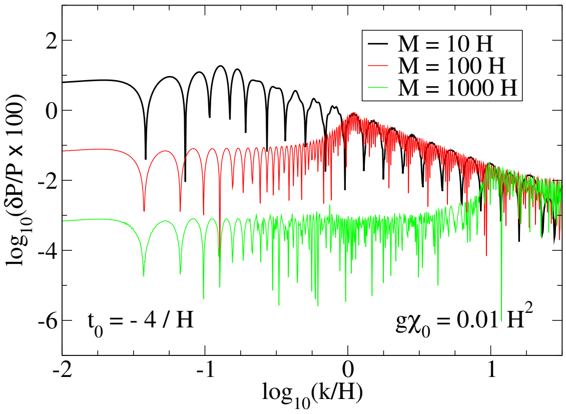

Figure 8: The log of the absolute value of the percent deviation in the power spectrum versus $\log(k/H)$ for $t_0 = -4/H$

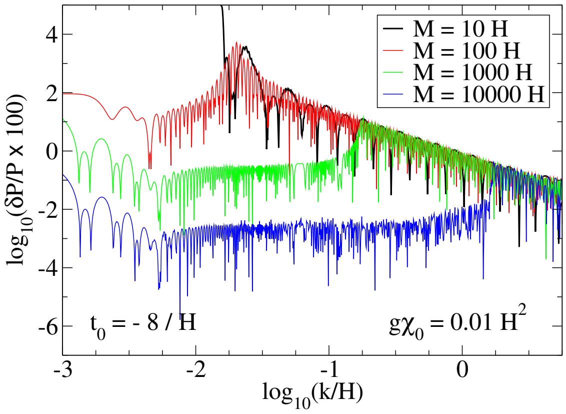

Figure 9: The same as in (a) but for $t_0 = -8/H$. Note that the curve for $M = 10H$ goes off-scale for low k and overlaps with the other curves for high k.

There is a simple intuitive understanding of the above behavior based on conservation laws and on how amplitudes and wave-numbers are damped during inflation. The pumping of ϕ by the oscillating χ field preferentially creates ϕ modes with physical wave numbers which are equal to $M/2$, although the wave-number is subsequently redshifted. The strongest production occurs for modes which are pumped right at the beginning of inflation since the χ amplitude is largest here, not having yet been damped as inflation proceeds. This explains both the value of $k_{\max}$ in (4.9) and the $k^{3/2}$ scaling of $\delta P/P$ at large k. The strongest deviations are initially created with $k/a(t_0) = M/2$, in agreement with (4.9). Perturbations which are created at later times $t > t_0$ then correspond to wave-number $k = a(t)M/2 > k_{\max}$, and the effect of χ on these perturbations is smaller by the damping factor $[a(t_0)/a(t)]^{3/2} \sim k^{-3/2}$. Furthermore, we can understand why the deviations also have power on scales $k < k_{\max}$: the pumping amplitude is proportional to the Fourier transform of the background χ field. This has an envelope $\sim e^{-3Ht/2}$ in addition to the oscillations. One therefore expects to see a spectrum of pairs with $k/a(t_0) \lesssim H$.

4.1 Phenomenology of trilinear coupling model

With the results of the previous section in hand, we can now address in more detail what range of parameters could give rise to a potentially observable effect on the CMB. Since we have already examined what choices for H, m and v maximize the range of pre-horizon-exit inflation to which the CMB is sensitive, in this section we instead focus on the $g'\chi\phi^2$ model.

To interpret our results, recall that the scale factor is normalized such that $a(0) = 1$; hence k is the physical wave number at $t = 0$. The value $k = H$ corresponds to the mode which is crossing the horizon at that instant. Since we have not specified how much inflation takes place after $t = 0$, we are free to imagine that this is the value of k which corresponds to some observable scale of interest, call it $k_{\rm COBE}$. We are interested in distortions of the spectrum of perturbations in this region. The most fortuitous situation would be that in which $k_{\rm COBE} = k_{\max}$, so that the deviations from the Harrison-Zeldovich spectrum would drop off for larger or smaller values of k, rather than increase to potentially ruled-out values on one side or the other. In what follows, let us focus on this special case, and consider how much fine tuning of the model parameters would be needed to get an observable effect.

For the $g'\chi\phi^2$ model, we find the following estimates. Since we are now assuming that $k_{\rm COBE} = k_{\rm max} = H$, eq. (4.9) implies that $M = 2He^{H|t_0|}$, and eq. (4.10) then gives $\frac{\delta P}{P}(k_{\rm max}) = g'A_0 H^{-2}$. Let us suppose that a 1% effect is observable; hence $g'A_0 = 0.01H^2$. Recall the bound that the initial χ energy density must be less than that of the inflaton: $M^2A_0^2e^{3H|t_0|} \lesssim H^2M_p^2$. Eliminating M and A_0, we obtain a bound on the number of e-foldings which can have occurred prior to horizon-crossing of the COBE-scale mode:

$$H|t_0| \lesssim \frac{1}{2}\ln\left(\frac{10^4 g'^2 M_p^2}{4H^4}\right)^{1/5} \lesssim \frac{4}{5}\ln\frac{10M_p}{H} \sim 33\,. \tag{4.11}$$

The numerical result 33 is based on taking $g' \sim M_p$ and a conservative value $H = 100\,\text{GeV}$ for the Hubble parameter during inflation. Such a value can be obtained for natural choices of parameters in hybrid inflation [8]. With some tuning, a lower value of H could be obtained. For consistency, we must check that the rate of χ damping due to ϕ production does not exceed the Hubble damping rate. The former can be estimated as the single particle decay rate for $\chi \to 2\phi$, which is $\Gamma = \frac{g'^2}{16\pi M}$. Demanding that $\Gamma < H$ and again using $M = 2He^{H|t_0|}$, we find that $g' < \sqrt{32\pi}He^{H|t_0|/2} \sim 3\times 10^{19}\,\text{GeV}$, consistent with our assumptions.

This verifies our expectation that the $g'\chi\phi^2$ model is sensitive to χ oscillations over more e-foldings than is possible for the $g\chi^2\phi^2$ hybrid-inflation model. In so doing it also underlines our observation that trans-Planckian effects could be mimicked without having to tune the duration of inflation so that COBE scales crossed the horizon exactly at the beginning of inflation. An interesting feature of the choice $g' = M_p$ is that the effect would be caused by relatively small initial amplitude oscillations of χ, with $A_0e^{3H|t_0|} \sim 10^4\,\text{GeV}$. On the other hand, this requires a small value of the inflaton itself, $\phi < MH/M_p = 0.1\,\text{GeV}$, to insure that the inflaton rolls slowly. This again requires the use of a small-field inflation model like hybrid inflation [16].

To give a better idea of what the actual observable effects are, we have used our distorted initial power spectra as inputs to the CMBFAST [18] program, to produce sample plots of the power spectrum of CMB temperature fluctuations. Figure 10 show how a given template for the initial power spectrum, translates into temperature fluctuations. This template was produced using the parameters $t_0 = -8/H$, $M = 1000H$ and a large amplitude $g'A_0 = 0.1$, chosen to make the effects more visible. The different choices for $k_{\rm COBE}$ (in Mpc^{-1}) correspond to deciding which physical scale to associate with a given feature in the input spectrum (controlled by the number of e-foldings between the horizon crossing of that mode and the end of inflation). We have normalized the different curves so that they all have the same magnitude at the first Doppler peak. The fast oscillations of the input power are washed out in the temperature fluctuations, but the low frequency modulations have a clearly visible effect.

The above examples used an unrealistically large distortion of the primordial power spectrum in order to illustrate the possibilities qualitatively. For a more realistic example,

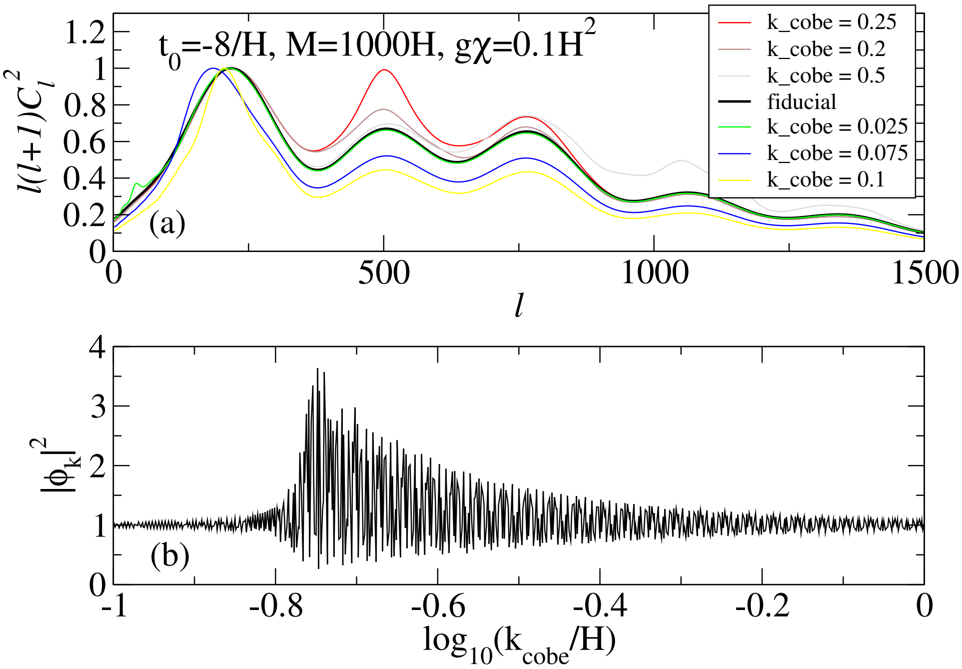

Figure 10: Top: Effect of modulations of the primordial power spectrum on the CMB temperature fluctuations. The order of curves in the legend coincides with the order in the second Doppler peak. Bottom: The modulated input power spectrum. The scale $k_{\rm COBE}$ is measured in units of $\rm Mpc^{-1}$.

we consider the case of figure 11 in which the observable effects are at the 5% level. To get this large a deviation in the CMB, it is necessary to have a considerably larger deviation (30%) in $|\tilde{\phi}_k^+(\infty)|^2$, to compensate for the fact that the oscillatory behavior of the latter tends to wash out its effects. The effect of shifting the value of $k_{\rm COBE}$ is clearly visible — it shifts the multipole values where the distortion in the CMB is most pronounced. In particular, these examples show that it is possible to avoid any observable deviation at low l.

4.2 Pre-inflationary oscillations

In the previous subsections we took the oscillations of the pumping field χ to begin at the same time as the inflationary phase. We will now show that qualitatively different results can be obtained by having the oscillations start prior to the beginning of inflation. In particular the χ oscillations could initially dominate the energy density of the universe, so that inflation is preceded by a matter-dominated period.

The transition from matter to inflaton domination can be modelled by a scale factor with the behavior

$$a(t) = \begin{cases} (t/t_1)^{2/3}, & t < t_1 \\ e^{2t/3t_1 - 2/3}, & t > t_1 \end{cases} = \begin{cases} \eta^2/4, & \eta < 2 \\ 1/(3-\eta), & 2 < \eta < 3 \end{cases} \tag{4.12}$$

Here we have introduced conformal time for convenience, defined by $d\eta = 2/(3t_1 a)d\eta$. The

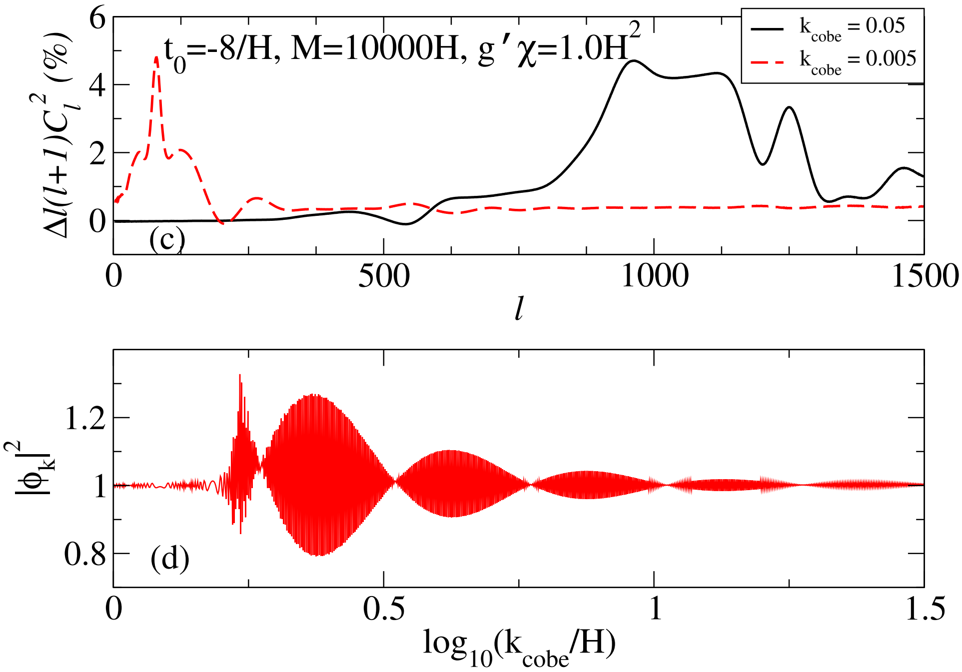

Figure 11: Top: Percent deviation in the CMB temperature anisotropy due to χ field oscillations for a given choice of model parameters. Bottom: The underlying modulated input power spectrum. $k_{\rm cobe}$ is in units of Mpc^{-1}.

solution for the oscillating scalar continues to have the form

$$\chi(t) = \chi_0 \cos[M(t-t_0)] \left(\frac{a(t_0)}{a(t)}\right)^{3/2} . \tag{4.13}$$

despite the change in the functional form of $a(t)$ at $t = t_i$.

However in this situation, there is an interesting effect on the inflaton fluctuations even in the absence of the pumping scalar, due to the time variation of the background geometry. The solutions for the fluctuations have different forms in the matter and inflaton dominated eras:

$$\begin{aligned} \phi_k(\eta) &= A_1 \left(3\,\frac{\eta\, k\cos{(\eta\, k)} - \sin{(\eta\, k)}}{\eta^4} + \frac{k^2\sin{(\eta\, k)}}{\eta^2}\right) \\ &+ A_2 \left(-3\,\frac{\eta\, k\sin{(\eta\, k)} + \cos{(\eta\, k)}}{\eta^4} + \frac{k^2\cos{(\eta\, k)}}{\eta^2}\right), \qquad \eta < 2 \end{aligned} \tag{4.14}$$

and

$$\begin{aligned} \phi_k(\eta) &= B_1\ (k\,(-3+\eta)\cos{(\eta\, k)} - \sin{(\eta\, k)}) \\ &+ B_2\ (k\,(-3+\eta)\sin{(\eta\, k)} + \cos{(\eta\, k)})\,, \qquad \eta > 2 \end{aligned} \tag{4.15}$$

It is obvious that if we form positive and negative frequency combinations of the independent solutions in each era, there will be mixing of the two kinds of solutions when we match the functions at $\eta = 2$. Therefore if we start in the vacuum state appropriate for the

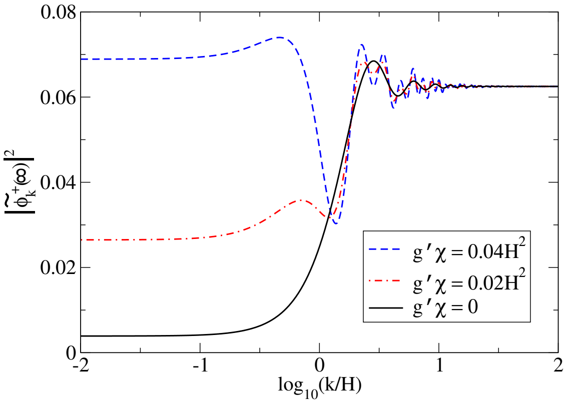

Figure 12: $|\tilde{\phi}_k(\infty)|^2$ as a function of $\log_{10}(k/H)$ in the trilinear coupling model, when the universe is initially dominated by the χ field oscillations.

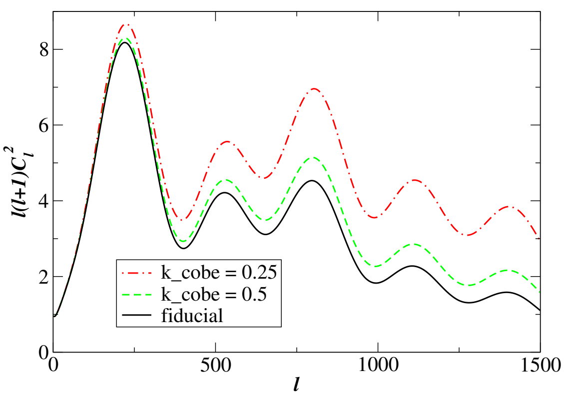

Figure 13: CMB temperature anisotropies for the $g' = 0$ case, compared to the standard best fit cosmological model.

matter dominated era, the inflaton will not be in the usual Bunch-Davies vacuum when it enters the inflationary era. This phenomenon was first noticed by Ford and Vilenkin [19]. The only scale associated with the spacetime background is the value of the Hubble rate at the transition time; thus one expects that modes with $k \lesssim H$ will be affected, and there will be no distortion of the spectrum for $k \gg H$.

This is precisely the kind of behavior we see when we numerically integrate to follow the scalar field evolution in the case where there is no coupling of ϕ to an external field χ, starting with initial conditions appropriate for the purely positive frequency state of $\phi_k(\eta)$. (We also checked that the analytic solution given above agrees with our numerical results.) The spectral distortion can be clearly seen in the $g'\chi = 0$ curve in figure 12, where the amplitude of $\phi_k(\infty)$ is much smaller for $k \ll H$ than for $k \gg H$. The sign of the effect is surprising; one might have expected particle production of low-k modes to have produced the opposite effect.

As before, by adjusting the total duration of inflation we can shift the effect of the distortion to lower or higher multipole values in the CMB, as illustrated in figure 13.

When we turn on the coupling g', we see from figure 12 that it is possible to counteract the suppression of power at low k. However it would require some fine tuning of g' to arrange for this compensation to be exact. In any case, the new features in the spectrum occur for k values in the range $k \lesssim H$. The only way these can show up in the CMB is if the transition between matter and radiation domination occurs just when the relevant modes cross the horizon. Thus we are back to the situation where the amount of inflation is the bare minimum compatible with solving the horizon and flatness problems.

This is in contrast to the previous case where horizon crossing could occur some time after the beginning of inflation by adjusting the χ mass M to larger values. The difference is that, similarly to the standard hybrid inflation model, in the present case the largest distortions of power always occur at small wave numbers. Although there is a peak in the spectrum at high k given by $Me^{-H|t_i|}/2$, it is only a local maximum, and never exceeds the

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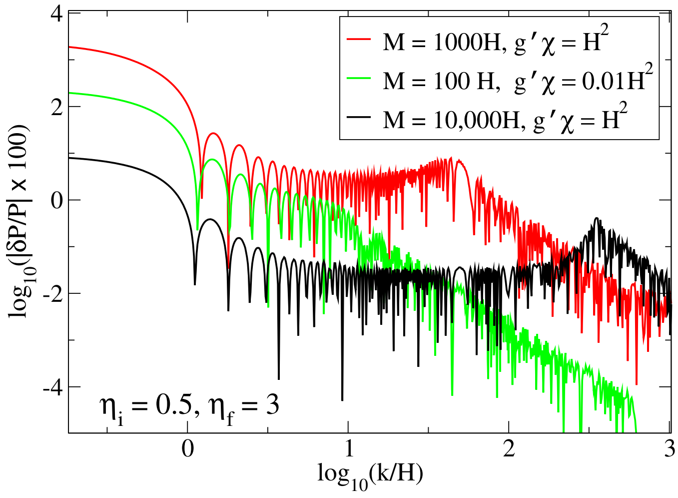

Figure 14: Log of the percent deviation in the power spectrum versus $\log(k/H)$ when inflation is preceded by a matter dominated period. Order of legends is the same as that of curves at the left hand side of the graph.

size of the deviation that occurs at low k values. This is the essential difference between the present situation and the previous case where the evolution was presumed to start during inflation. To reiterate, the difference arises because the prior period of matter domination puts the inflaton in a state which is not the same as the usually presumed Bunch-Davies vacuum.

5. Conclusions

Clearly, using the CMB to probe energy scales so far beyond what could be done in accelerators in an intriguing possibility. But, at least in principle, this approach carries the risk of completely undermining the predictiveness of inflationary models, and so negating the value of comparing them with CMB observations.

What our results show is that there can be a reasonable window onto higher-energy physics purely within a model under which all calculations are completely under quantitative control. The window we find is onto any epoch of inflation prior to horizon exit but after inflation begins, provided that this epoch is not too long (i.e. not longer than from 10 to 30 e-foldings, depending on the model). A longer pre-horizon epoch of inflation would be sufficient to completely remove the effects for the CMB of any early oscillating scalar modes.

Indeed, it is not too surprising that some sensitivity exists within the CMB towards the cosmic expansion just before horizon exit, since the standard inflationary predictions relate the CMB fluctuations to the size of the slow-roll parameters at horizon exit. For instance, if observed scales were to exit the horizon just as inflation began, then even standard calculations would not predict a spectral index near unity. What our calculation does is to extend this sensitivity to many more e-foldings of pre-horizon-exit inflation provided that the non-adiabatic scalar oscillations we consider are present during these times.

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Three features of our results bear particular emphasis.

- First, they allow for new effects in the CMB, but do not make them generic. This means that the standard predictions are very robust to higher-energy physics, just as they had been expected to be. But our results also encourage the search for deviations from standard predictions, since they broaden the kinds of physics which can plausibly be expected to produce them.
- Second, our results do not contradict any of the well-established implications of decoupling. The kinds of oscillations we consider would not be expected to decouple by observers at any energy under consideration, since they can continually pump energy $E \sim M$ into the lower-mass modes. All observers therefore understand why it would be wrong to try to integrate out the heavy fields having mass M in the usual way, since modes of energy M are constantly being pumped by the scalar oscillations.
- Third, it is clear from our calculations — as well as from older calculations [19] — that for low wave-numbers the α vacua of de-Sitter space do make physical sense, since they may be obtained by evolving the usual FRW vacua forward in time in a non-adiabatic way. α-vacua can be expected to be obtained in this way up to a maximum wave-number, $k_{\rm max}$, whose value is set by the most energetic states which can be excited by the non-adiabatic evolution. Recent objections in principle [9] to using α vacua may in this way be seen to be traceable to the use of these vacua up to arbitrarily high wave-numbers.

Our calculation does not directly preclude the possibility that trans-Planckian physics might yet contribute uncontrollably to inflationary predictions to the CMB, since we have restricted our focus entirely to sub-Planckian physics for which calculations can be systematically made. Indeed it is likely to remain difficult to draw any definitive conclusions on this score without having a detailed understanding of the nature of trans-Planckian physics.

However, we believe our calculation *does* a new features to the trans-Planckian debate. First, it identifies non-adiabaticity as a key ingredient of previous examples of how trans-Planckian effects can influence the CMB. (This is in agreement with the results of Martin and Brandenberger [1], but by disentangling the adiabaticity from more exotic possibilities like non-Lorentz-invariant dispersion relations we also show that these exotic assumptions are not crucial for obtaining an observable effect.) The relevance of adiabaticity can be traced to its role in preventing the heavy physics from being integrated out despite the modes involved being heavy. From this point of view we believe that if trans-Planckian physics obeys our usual notions of decoupling (as, for example, string theory appears to do) then it is unlikely that it can affect the CMB unless it introduces new low-energy states or non-adiabatic physics at the scale of horizon exit.

Second, in the event that controlled calculations of trans-Planckian physics should ultimately be possible, our calculation provides a useful benchmark against which they may be compared. Indeed such a comparison would be necessary in order to distinguish really new trans-Planckian physics from the more mundane types which we consider here.

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In particular, our calculation shows that it is always possible to mock up an observed modulation of the observed CMB power spectrum in terms of some sort of α-vacuum, so in itself the power spectrum cannot be used to tell if any observed deviations are due to an α-vacuum or some other physics.

At any rate, it is probably safe to say that inflation offers the best hope to find relics of very-high-energy physics at the present time, be it trans-Planckian or not.

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